Dielectric function and electrical dc conductivity of nonideal plasmas

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(Received 2 October 1997; revised manuscript received 9 December 1997)

Within generalized linear response theory, an expression for the dielectric function is derived that is consistent with standard approaches to the electrical dc conductivity. Explicit results are given for the first moment Born approximation. Some exact relations as well as the limiting behavior at small values of wave number and frequency are investigated. [S1063-651X(98)06504-0]

PACS number(s): 52.25.Mq

I. INTRODUCTION

The dielectric function $\epsilon(\vec{k},\omega)$ describing the response of a charged particle system to an external, time and space dependent electric field (wave vector $\vec{k}$, frequency ω) is related to various phenomena such as electric conductivity and optical absorption of light. In particular, it is an important quantity for plasma diagnostics, see, e.g., recent applications to determine the parameters of high-density plasmas produced by picosecond lasers [1]. However, the application of widely used simplified expressions for the dielectric function is questionable in the case of nonideal plasmas.

As is well known, the electrical dc conductivity of a charged particle system should be obtained as a limiting case of the dielectric function. However, at present both quantities are treated by different theories. A standard approach to the electrical dc conductivity is given by the Chapman-Enskog approach [2]. In dense plasmas, where many-particle effects are of importance, linear response theory has been worked out to relate the conductivity to equilibrium correlation functions which can be evaluated using the method of thermodynamic Green functions, see [3]. This way it is possible to derive results for the conductivity of partially ionized plasmas not only on the level of ordinary kinetic theory, but also including two-particle nonequilibrium correlations [4].

On the other hand, the dielectric function can also be expressed in terms of equilibrium correlation functions. Neglecting collisions, the well-known random phase approximation (RPA, see also below) is obtained where the contribution of charged particles with mass m to the imaginary part of the dielectric function is proportional to $\omega k^{-3}\,\exp[-m\omega^2/(2k_BTk^2)]$. Obviously, a systematic perturbation expansion to include collision effects is difficult to carry out near the singular point $\vec{k}=\vec{0}$, $\omega=0$. Different improvements are known to go beyond the well-known RPA result. In the static limit, local field corrections have been discussed extensively [5], and the dynamical behavior of the corrections to the RPA in the long-wavelength limit was investigated in time-dependent mean field theory neglecting damping effects [6], see also [7] for the strong coupling case. At arbitrary $\vec{k}$ and ω, approximations are made on the basis of sum rules for the lowest moments [8]. However, these approximations cannot give an unambiguous expression for $\epsilon(\vec{k},\omega)$ in the entire $(\vec{k},\omega)$ space.

We shall give here a unified approach to the dielectric function as well as the dc conductivity, which is consistent with the Chapman-Enskog approach to dc conductivity and allows for a perturbation expansion also in the region of small $\vec{k}$ and ω. In the following Sec. II the method of generalized linear response [9] is presented which can be used to find very general relations between a dissipative quantity and correlation functions describing the dynamical behavior of fluctuations in equilibrium. A special expression for the dielectric function that is related to the use of the force-force correlation function in evaluating the dc conductivity is given.

Different methods can be applied to evaluate equilibrium correlation functions for nonideal plasmas. We shall use perturbation theory to evaluate thermodynamic Green functions [10]. Results in the Born approximation are given in Sec. III. Within a more sophisticated approach, partial summations could be performed using diagram techniques, as shown in Ref. [3]. To evaluate equilibrium correlation functions in strongly coupled plasmas, a promising alternative is given by molecular dynamics simulations. It is expected that reliable results for the dielectric function for dense systems by quantum molecular dynamics will be available in the near future. Work in this direction is in progress but will not be discussed in this paper.

To illustrate the general approach, explicit results for the dielectric function in the first moment Born approximation are given for hydrogen plasmas in Sec. IV. Sum rules as well as the dc conductivity are discussed. The simple approximation considered here will be improved in a subsequent paper [11], where a four-moment approach to two-component plasmas is investigated.

II. DIELECTRIC FUNCTION WITHIN GENERALIZED LINEAR RESPONSE THEORY

We consider a charged particle system consisting of different components with masses m_c and charges e_c. In the following we shall use the index c not only to denote species (e.g., electron e, ion i) but also to describe further internal degrees of freedom such as spin.

The charged particle system is investigated under the influence of an external potential $U_{\rm ext}(\vec{r},t) = e^{i(\vec{k}\cdot\vec{r}-\omega t)}U_{\rm ext}(\vec{k},\omega)+\text{c.c.}$ The total Hamiltonian $H_{\rm tot}(t) = H+H_{\rm ext}(t)$ contains the system Hamiltonian H and the interaction with the external potential

$$H_{\rm ext}(t)=U_{\rm ext}(\vec{k},\omega)e^{-i\omega t}\sum_{c,p} e_c n^c_{p,-k}+\text{c.c.}, \tag{1}$$

where

$$n^c_{p,k}=(n^c_{p,-k})^\dagger=a^\dagger_{c,p-k/2}a_{c,p+k/2} \tag{2}$$

is the Wigner transform of the single-particle density given in terms of creation and annihilation operators in the momentum representation, c indicating species (such as electron e, ion i) and spin.

Under the influence of the external potential, a time-dependent charge density

$$\begin{aligned}\frac{1}{\Omega_0}\sum_{c,p,k'} e_c\langle\delta n^c_{p,k'}\rangle^t e^{i\vec{k}'\cdot\vec{r}}+\text{c.c.}\\ =\frac{1}{\Omega_0}\sum_{c,p} e_c\delta f_c(\vec{p};\vec{k},\omega)e^{i(\vec{k}\cdot\vec{r}-\omega t)}+\text{c.c.}\end{aligned} \tag{3}$$

will be induced. Here, $\delta n^c_{p,k'}=n^c_{p,k'}-\text{Tr}\{n^c_{p,k'}\rho_0\}$ denotes the deviation from equilibrium, where the equilibrium statistical operator is given by

$$\rho_0=\frac{\exp\left(-\beta H+\beta\sum_c \mu_c N_c\right)}{\text{Tr}\,\exp\left(-\beta H+\beta\sum_c \mu_c N_c\right)}. \tag{4}$$

The average $\langle\cdots\rangle^t=\text{Tr}\{\cdots\rho(t)\}$ has to be performed with the nonequilibrium statistical operator $\rho(t)$, which is derived in linear response with respect to the external potential in Appendix A. For homogeneous and isotropic systems, we find simple algebraic relations between the different modes $(\vec{k},\omega)$ of the external potential $U_{\rm ext}(\vec{k},\omega)$ and the induced single-particle distribution

$$\delta f_c(\vec{p};\vec{k},\omega)=e^{i\omega t}\langle\delta n^c_{p,k}\rangle^t, \tag{5}$$

which allows one to introduce the dielectric function $\epsilon(k,\omega)$, the electric conductivity $\sigma(k,\omega)$, and the polarization function $\Pi(k,\omega)$. From standard electrodynamics we have

$$\epsilon(k,\omega)=1+\frac{i}{\epsilon_0\omega}\sigma(k,\omega)=1-\frac{1}{\epsilon_0 k^2}\Pi(k,\omega) \tag{6}$$

and

$$\Pi(k,\omega)=\frac{1}{\Omega_0}\sum_{c,p} e_c\delta f_c(\vec{p};\vec{k},\omega)\frac{1}{U_{\rm eff}(k,\omega)}, \tag{7}$$

where the polarization function is defined with respect to the effective potential

$$U_{\rm eff}(k,\omega)=U_{\rm ext}(k,\omega)/\epsilon(k,\omega). \tag{8}$$

Using the equation of continuity

$$\omega\sum_p \delta f_c(\vec{p};\vec{k},\omega)=\frac{k}{m_c}\sum_p \hbar p_z\delta f_c(\vec{p};\vec{k},\omega), \tag{9}$$

where the z direction is parallel to $\vec{k}$, $\vec{k}=k\vec{e}_z$, we can also express

$$\begin{aligned}\Pi(k,\omega)&=\frac{k}{\omega}\frac{1}{\Omega_0}\sum_{c,p}\frac{e_c}{m_c}\hbar p_z\delta f_c(\vec{p};\vec{k},\omega)\frac{1}{U_{\rm eff}(k,\omega)}\\ &=\frac{k}{\omega}\langle J_k\rangle^t e^{i\omega t}\frac{1}{U_{\rm eff}(k,\omega)}\end{aligned} \tag{10}$$

with the current density operator

$$J_k=\frac{1}{\Omega_0}\sum_{c,p}\frac{e_c}{m_c}\hbar p_z n^c_{p,k}, \tag{11}$$

Ω_0 is the normalization volume.

The main problem in evaluating the mean value $\langle J_k\rangle^t$, Eq. (11), of the current density is the determination of $\rho(t)$. In linear response theory where the external potential is considered to be weak, the statistical operator $\rho(t)$ up to first order in $U_{\rm ext}(k,\omega)$ can be given explicitly, see Appendix A. An important ingredient to generalized linear response theory is that a set of relevant observables can be introduced whose mean values characterize the nonequilibrium state of the system. In this paper, we shall consider the current density J_k, Eq. (11), as a relevant observable. This observable corresponds to the first moment of the single-particle distribution function. The extension to more general sets of relevant observables such as higher moments of the distribution function is discussed in Appendix A.

Based on this first moment approach, we have the following expression for the polarization function:

$$\Pi(k,\omega)=-\epsilon(k,\omega)\frac{ik^2\beta\Omega_0}{\omega}\frac{(J_k;J_k)^2}{M_{JJ}}, \tag{12}$$

with

$$\begin{aligned}M_{JJ}=&-i\omega(J_k;J_k)+\langle\dot{J}_k;\dot{J}_k\rangle_{\omega+i\eta}\\ &-\frac{\langle\dot{J}_k;J_k\rangle_{\omega+i\eta}}{\langle J_k;J_k\rangle_{\omega+i\eta}}\langle J_k;\dot{J}_k\rangle_{\omega+i\eta}.\end{aligned} \tag{13}$$

The equilibrium correlation functions are defined as

$$\begin{aligned}(A;B)=(B^\dagger;A^\dagger)&=\frac{1}{\beta}\int_0^\beta d\tau\,\text{Tr}[A(-i\hbar\tau)B^\dagger\rho_0],\\ \langle A;B\rangle_z&=\int_0^\infty dt e^{izt}(A(t);B),\end{aligned} \tag{14}$$

with $A(t)=\exp(iHt/\hbar)A\exp(-iHt/\hbar)$, and $\dot{A}=i[H,A]/\hbar$.

Before evaluating the polarization function (12) for a two-component plasma, we shall first discuss its relation to the Kubo formula and afterwards the significance of the dielectric function $\epsilon(k,\omega)$ occurring in Eq. (12).

Applying partial integration

$$\langle A;B\rangle_z=\frac{i}{z}[(A;B)+\langle\dot{A};B\rangle_z]=\frac{i}{z}[(A;B)-\langle A;\dot{B}\rangle_z], \tag{15}$$

the time derivatives in the time correlation functions of expression (13) can be eliminated,

$$M_{JJ}=-\eta(J_k;J_k)+(\dot{J}_k;J_k)+(J_k;J_k)^2/\langle J_k;J_k\rangle_{\omega+i\eta}. \tag{16}$$

Furthermore, we use the property

$$(\dot{A};B)=\frac{i}{\hbar\beta}\,\mathrm{Tr}\{[A,B^\dagger]\rho_0\} \tag{17}$$

[to prove this, perform the integral in the definition (14)], so that $(\dot{J}_k;J_k)=i\ \mathrm{Tr}\{[J_k,J_{-k}]\rho_0\}/(\hbar\beta)=0$. In conclusion, expression (12) for the polarization function can be rewritten as

$$\Pi(k,\omega)=-\epsilon(k,\omega)\frac{ik^2\beta\Omega_0}{\omega}\frac{(J_k;J_k)\langle J_k;J_k\rangle_{\omega+i\eta}}{(J_k;J_k)-\eta\langle J_k;J_k\rangle_{\omega+i\eta}}. \tag{18}$$

Performing the limit $\eta\to0$, we obtain for finite values of the correlation function $\langle J_k;J_k\rangle_{\omega+i\eta}$ the simple result

$$\Pi(k,\omega)=-\epsilon(k,\omega)\frac{ik^2\beta\Omega_0}{\omega}\langle J_k;J_k\rangle_{\omega+i\eta}, \tag{19}$$

which is denoted as the Kubo formula for the polarization function. Similarly, this result can also be obtained from more general sets of observables. In particular, the Kubo formula can be derived if the set of relevant observables is empty, see Appendix A, Eq. (A15). Different approaches based on different sets of relevant observables are formally equivalent as long as no approximations in evaluating the correlation functions are made.

However, expressions (12) and (19) are differently suited to perform perturbation expansions. For this we consider the dc conductivity

$$\sigma=\lim_{\omega\to0}\lim_{k\to0}\sigma(k,\omega)=i\lim_{\omega\to0}\lim_{k\to0}\frac{\omega}{k^2}\Pi(k,\omega). \tag{20}$$

We compare the correlation function

$$\beta\Omega_0\langle J_0;J_0\rangle_{i\eta} \tag{21}$$

in the Kubo formula (19) with the expression

$$\beta\Omega_0\frac{(J_0;J_0)^2}{\langle\dot{J}_0;\dot{J}_0\rangle_{i\eta}-\langle\dot{J}_0;J_0\rangle_{i\eta}\langle J_0;J_0\rangle_{i\eta}^{-1}\langle J_0;\dot{J}_0\rangle_{i\eta}}, \tag{22}$$

arising in the corresponding formula (12) [as discussed at the end of this section, the prefactor $\epsilon(k,\omega)$ disappears if only irreducible contributions of the correlation functions are considered]. It is evident that perturbation theory cannot be applied to expression (21) because in zeroth order this expression is already diverging. In contrast, expression (22) allows for a perturbative expansion. The denominator vanishes in zeroth order in the interaction. The correlation function $\langle\dot{J}_0;\dot{J}_0\rangle_{i\eta}$ gives a contribution already of second order in the interaction, whereas the remaining part contributes only from fourth order on. For instance, in the Born approximation the Faber-Ziman result for the electric conductivity is obtained [3]. The expression $\sigma^{-1}\sim\langle\dot{J}_0;\dot{J}_0\rangle_{i\eta}$ is also known as the force-force correlation function expression for the resistivity. More precisely, the resistivity should be given in terms of stochastic forces which are related to the second term in the denominator of Eq. (22), see also Eq. (A12) in Appendix A. The applicability of correlation functions for the inverse transport coefficients has been widely discussed, for a review see Ref. [9]. The approach to the dielectric function given in the present paper is based on the choice (11) for the set of relevant observables and may be considered as the generalization of the force-force correlation function method for the electric resistivity to the dielectric function. Possible extensions of the set of relevant observables have been investigated in evaluating the dc conductivity in Ref. [3] and will be considered in evaluating the dielectric function in a forthcoming paper [11].

The origin of the dielectric function $\epsilon(k,\omega)$ in Eq. (12) is due to the definition of the polarization function (screened susceptibility) with respect to the effective potential $U_{\mathrm{eff}}(k,\omega)$, Eq. (8). For instance, the Kubo formula (19) can be rewritten as

$$\frac{1}{\epsilon(k,\omega)}-1=-i\frac{\beta\Omega_0}{\epsilon_0\omega}\langle J_k;J_k\rangle_{\omega+i\eta}. \tag{23}$$

A similar relation can also be found for Eq. (12).

On the other hand, the occurrence of the dielectric function in the expressions for the polarization function has a simple consequence if the correlation functions are evaluated by standard many particle methods such as perturbation theory for thermodynamic Green functions. In this context the correlation functions containing $\epsilon(k,\omega)J_k$ are obtained from irreducible diagrams to Green functions containing J_k, which cannot be separated into two pieces by cutting a single interaction line.

III. EVALUATION OF CORRELATION FUNCTIONS

We apply the method developed above to two-component plasmas consisting of electrons (mass m_e, charge e_e, density n_e) and ions (mass m_i, charge e_i, density n_i) with $e_en_e+e_in_i=0$ for a charge-neutral plasma. The Hamiltonian is given by

$$H=\sum_{c,p}E_p^c a_{c,p}^\dagger a_{c,p}+\frac{1}{2}\sum_{cc',pp'q}V_{cc'}(q)a_{c,p-q}^\dagger a_{c',p'+q}^\dagger a_{c',p'}a_{c,p}, \tag{24}$$

where $E_p^c=\hbar^2p^2/2m_c$ denotes the kinetic energy and $V_{cc'}(q)=e_ce_{c'}/(\epsilon_0\Omega_0q^2)$ describes the Coulomb interaction between electrons and ions as well as the electron-electron and ion-ion interaction.

Within the generalized linear response approach, the polarization function is given in terms of correlation functions. The correlation functions occurring in Eq. (12) contain the operators $n_{p,k}^c=a_{c,p-k/2}^\dagger a_{c,p+k/2}$ and $\dot{n}_{p,k}^c=-(i\hbar p_zk/m_c)n_{p,k}^c$

$+v^c_{p,k}$, where the first term arises from the kinetic energy and gives a contribution even if the collisions in the plasma are neglected, i.e., to the RPA result. The second term

$$v^c_{p,k}=\frac{i}{\hbar}\sum_{c',p',q} V_{cc'}(q)[a^\dagger_{c,p-k/2-q}a^\dagger_{c',p'+q}a_{c',p'}a_{c,p+k/2} \\ -a^\dagger_{c,p-k/2}a^\dagger_{c',p'+q}a_{c',p'}a_{c,p+k/2+q}] \tag{25}$$

contains the interaction $V_{cc'}(q)$. It has to be taken into account if collisions are included.

To evaluate the correlation functions, we perform a perturbation expansion with respect to the interaction $V_{cc'}(q)$. Within a quantum statistical approach, the correlation functions are related to Green functions which can be evaluated by diagram techniques. This has been discussed in detail for the case of the static electric conductivity [3] and will not be detailed here any further. Instead, we will consider only the lowest orders of perturbation theory, see Appendix B.

As shown in Appendix B, evaluating the polarization function to zeroth order, the RPA result for the dielectric function is reproduced. Expanding up to second order with respect to $V_{cc'}(q)$,

$$\Pi(k,\omega)=-i\beta\Omega_0\frac{k^2}{\omega}\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta} \\ \times\left[1+\sum_{cd,pp'}\frac{\hbar^2}{\Omega_0^2}\frac{e_ce_d}{m_cm_d}p_zp'_z\frac{\langle v^c_{p,k};v^d_{p',k}\rangle^{(0)}_{\omega+i\eta}}{(J_k;J_k)^{(0)}}\right. \\ \times\left(\frac{1}{\eta-i\omega+i\frac{\hbar}{m_d}p'_zk}+\frac{1}{\eta-i\omega+i\frac{\hbar}{m_c}p_zk}\right. \\ \left.\left.-\frac{\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta}}{(J_k;J_k)^{(0)}}\right)\right]^{-1}, \tag{26}$$

collisions are included. The correlation functions $\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}$ are of at least second order in the interaction $V_{cc'}(q)$. Evaluating the correlation function containing creation and annihilation operators, cf. Eq. (25), to zeroth order in the interaction, the collisions are taken into account in the Born approximation. Note that the prefactor $\epsilon(k,\omega)$ disappears if only irreducible diagrams are considered in evaluating the correlation functions.

In the nondegenerate case the following expression is obtained:

$$\Pi(k,\omega)=-\beta\sum_c e_c^2n_c[1+z_cD(z_c)] \\ \times\left[1-i\frac{\omega}{k^2}\frac{e_e^2e_i^2}{(4\pi\epsilon_0)^2}n_en_i\frac{\mu_{ei}^{1/2}}{(k_BT)^{5/2}}\frac{2(2\pi)^{1/2}}{\sum_c e_c^2n_c/m_c}\right. \\ \left.\times\int_0^\infty dpe^{-p^2}\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right)W(p)\right]^{-1}, \tag{27}$$

with

$$W(p)=\frac{2}{3}p\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)^2\frac{\sum_c e_c^2n_c[1+z_cD(z_c)]}{\sum_c e_c^2n_c/m_c} \\ -\frac{M_{ei}^{1/2}}{\mu_{ei}^{1/2}}\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)\int_{-1}^1 dcc\left[e_eD\left(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp\right)\right. \\ \left.+e_iD\left(z_{ei}+\sqrt{\frac{m_e}{m_i}}cp\right)\right]. \tag{28}$$

Here, $z_{ei}=\omega/k\sqrt{M_{ei}/2k_BT}$, $z_c=\omega/k\sqrt{m_c/2k_BT}$, $\lambda(p)=(\hbar^2\kappa^2)/(4\mu_{ei}k_BTp^2)+1$, $M_{ei}=m_e+m_i$, $\mu_{ei}=m_em_i/M_{ei}$, and

$$D(z)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^\infty e^{-x^2}\frac{dx}{x-z-i\eta}=i\sqrt{\pi}e^{-z^2}[1+\mathrm{erf}(iz)] \tag{29}$$

denotes the Dawson integral. Equation (27) is an analytic expression for the polarization function which can be evaluated in the entire $(\vec{k},\omega)$ space. Note that a statically screened potential was used in Eq. (25) to obtain a convergent collision integral, the screening parameter is given by $\kappa^2=\Sigma_ce_c^2n_c/(\epsilon_0k_BT)$. From Eq. (27) it can be seen immediately that the RPA result is obtained in the limit of vanishing interactions, $W(p)=0$.

IV. RESULTS FOR HYDROGEN PLASMAS

Expression (27) for the polarization function is simplified for a system consisting of protons and electrons, where $e_i=-e_e$, $n_i=n_e$, and $m_i/m_e=1836$:

$$\epsilon(k,\omega)=1+\frac{e^2n}{\epsilon_0k_BTk^2}[2+z_eD(z_e)+z_iD(z_i)]\left[1-i\frac{\omega}{k^2}\frac{e^4}{(4\pi\epsilon_0)^2}n\frac{\mu_{ei}^{1/2}}{(k_BT)^{5/2}}2(2\pi)^{1/2}\int_0^\infty dpe^{-p^2}\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right)\right. \\ \left.\times\left\{\frac{2}{3}p[2+z_eD(z_e)+z_iD(z_i)]-\left(\frac{M_{ei}}{\mu_{ei}}\right)^{1/2}\int_{-1}^1dcc\left[D\left(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp\right)-D\left(z_{ei}+\sqrt{\frac{m_e}{m_i}}cp\right)\right]\right\}\right]^{-1}. \tag{30}$$

We first discuss the limiting case of small k. For $k \ll \omega\sqrt{m_e/(2k_BT)}$ we use the expansion

$$D(z)=i\sqrt{\pi}e^{-z^2}-\frac{1}{z}-\frac{1}{2z^3}\pm\cdots \tag{31}$$

so that after expansion also with respect to cp/z_{ei} we have

$$\epsilon(0,\omega)=1-\frac{\omega_{\rm pl}^2}{\omega^2+i\omega/\tau} \tag{32}$$

with $\omega_{\rm pl}^2=e^2n/(\epsilon_0\mu_{ei})$ and

$$\tau=\frac{(4\pi\epsilon_0)^2}{e^4}\frac{(k_BT)^{3/2}\mu_{ei}^{1/2}}{n}\frac{3}{4(2\pi)^{1/2}} \times\left[\int_0^\infty dp\,p e^{-p^2}\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right)\right]^{-1}. \tag{33}$$

According to Eq. (6), the dc conductivity

$$\sigma(0,\omega\to 0)=\omega_{\rm pl}^2\epsilon_0\tau \tag{34}$$

is obtained, which coincides with the Faber-Ziman formula at finite temperatures [3].

On the other hand, in the limiting case of small ω we use the expansion

$$D(z)=i\sqrt{\pi}e^{-z^2}-2z+\frac{4}{3}z^3\pm\cdots \tag{35}$$

for $\omega \ll \sqrt{2k_BT/m_i}k$ and obtain

$$\lim_{k\to 0}\lim_{\omega\to 0}\epsilon(k,\omega)=1+\frac{\kappa^2 d}{-i\omega+dk^2}\left(1+i\frac{\omega}{2k}\sqrt{\frac{\pi m_i}{2k_BT}}\right), \tag{36}$$

with

$$d^{-1}=-\frac{e^4}{(4\pi\epsilon_0)^2}n\frac{4(2\pi)^{1/2}\mu_{ei}^{1/2}}{(k_BT)^{5/2}}\int_0^\infty\frac{dp}{p}e^{-p^2} \times\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right). \tag{37}$$

Here, in evaluating the last expression of Eq. (30), also $z_{ei}+(m_e/m_i)^{1/2}cp$ is considered as a small quantity, whereas $z_{ei}-(m_i/m_e)^{1/2}cp$ is large in the region of relevant p. For small values $k<2\sqrt{2k_BT/(\pi m_i)}/d$, the second term in the numerator of Eq. (36) can be neglected, and the diffusion type form of $\epsilon(k,\omega)$ is obtained, see [12].

As an example, a dense plasma is considered with parameter values $T=50$ eV and $n_e=3.2\times10^{23}$ cm^{-3}. Such parameter values have been reported recently in laser produced high-density plasmas by Sauerbrey *et al.*, see [1]. We will use Rydberg units so that $T=3.68$ in Ry and $n_e=0.0474$ in a_B^{-3}. At these parameter values, the plasma frequency is obtained as $\omega_{\rm pl}=1.54$, and the screening parameter as $\kappa=0.805$.

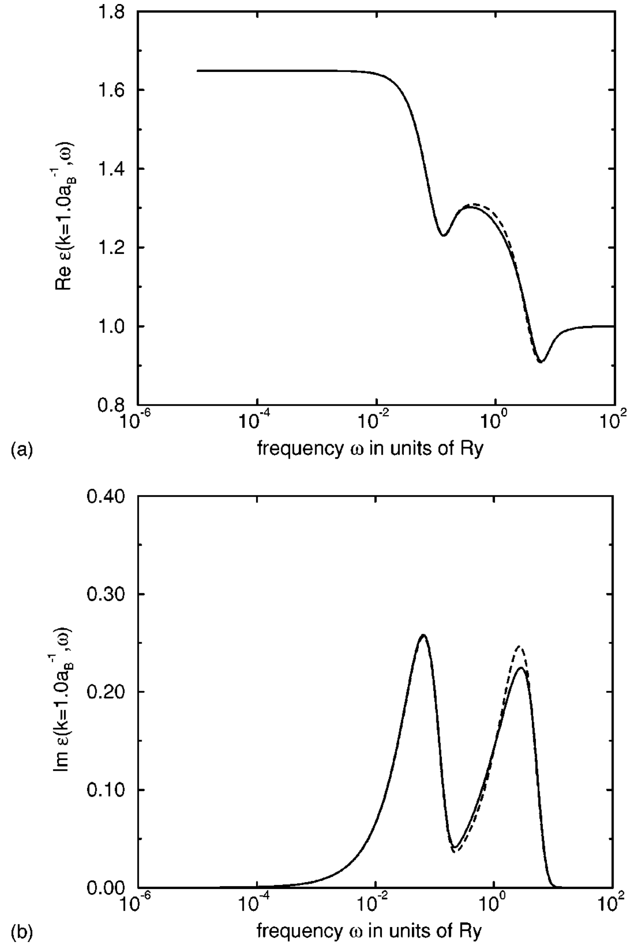

FIG. 1. $\epsilon(k,\omega)$ as a function of ω (in Ry/$\hbar$) at $k=1/a_B$ for a hydrogen plasma, $n_e=3.2\ 10^{23}$ cm^{-3}, $T=50$ eV. Upper panel, Re ϵ; lower panel, Im ϵ; broken line, RPA; full line, first moment Born approximation.

First we discuss the dependence of the dielectric function on frequency for different values of k, see Figs. 1–4. For large values of k our result for the dielectric function coincides with the RPA result. At decreasing k strong deviations are observed.

Both the RPA expression as well as the expression (30) for the dielectric function fulfill important relations such as the Kramers-Kronig relation and the condition of total screening. The validity of the sum rule

$$\int_0^\infty\omega\ \mathrm{Im}\ \epsilon(k,\omega)d\omega=\frac{\pi}{2}\omega_{\rm pl}^2 \tag{38}$$

is checked by numerical integration. The RPA result coincides with the exact value $\omega_{\rm pl}^2\pi/2=3.74$ to be compared with expression (30) which gives 3.74 at $k=1$, 3.75 at $k=0.1$, 3.71 at $k=0.01$, and 3.74 at $k=0.001$. The small deviations are possibly due to numerical accuracy.

To investigate the behavior at small k, we give a log-log plot of Im $\epsilon(k,\omega)$ as a function of ω for different values k in Fig. 5. For $\omega>\sqrt{2k_BT/m_e}k=3.84\,k$ the Drude-like behavior (32) is clearly seen, with $\tau=8.36$.

Considering the limit of small ω, a log-log plot of Im $\epsilon(k,\omega)$ as a function of k for different values ω is shown

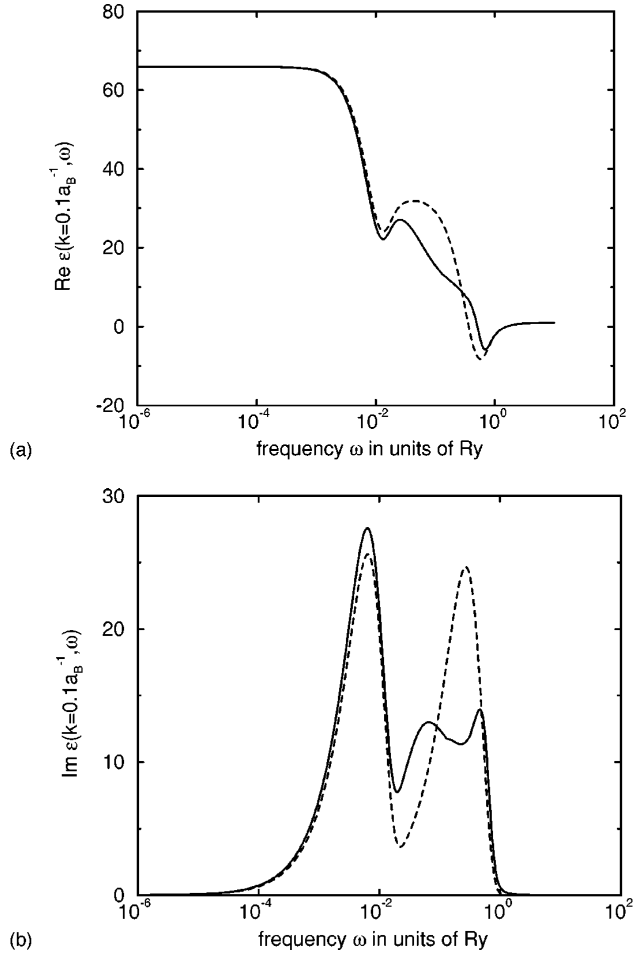

FIG. 2. The same as Fig. 1 for $k = 0.1/a_B$.

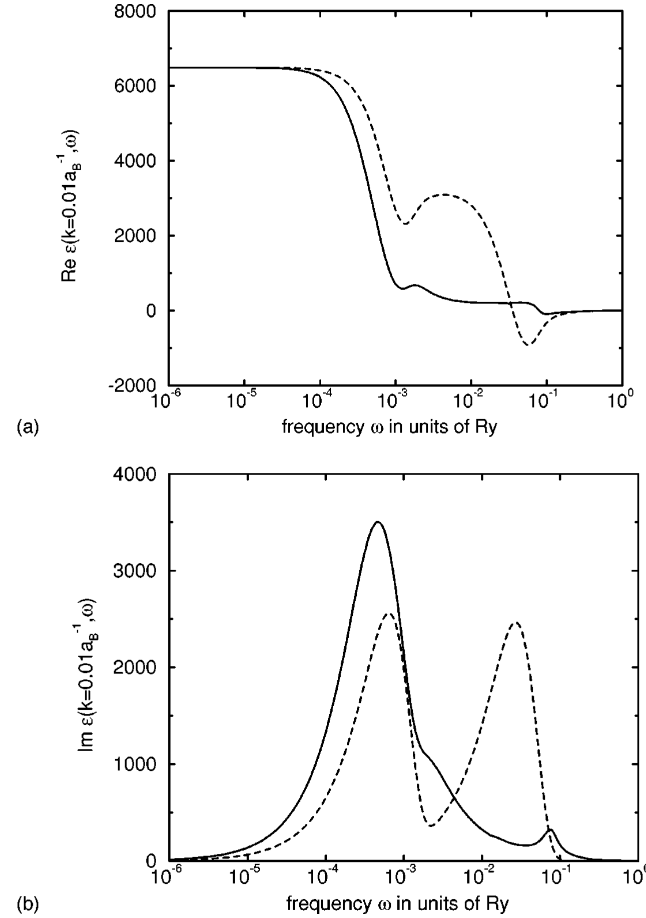

FIG. 3. The same as Fig. 1 for $k = 0.01/a_B$.

in Fig. 6. The diffusion behavior (36) occurs for $k < \sqrt{2k_BT/(\pi m_i)} = 0.00732$ at $k > \sqrt{m_i/(2k_BT)}\,\omega = 11.17\,\omega$ with $d = 13.8$. Altogether the numerical evaluation of the general expression (30) for the dielectric function confirms the validity of the simple limiting formulas (32) and (36).

In this paper we have focused on discussion of the properties only of $\epsilon(k,\omega)$. Related quantities such as $\epsilon^{-1}(k,\omega)$ will be investigated in a forthcoming paper [11]. The parameter values for density and temperature can be extended to other nondegenerate plasmas such as ordinary laboratory plasmas or the solar plasma. This has been done with results showing the same qualitative behavior of the expression (30), but at shifted values of k and ω.

V. CONCLUSIONS

An expression for the dielectric function of Coulomb systems is derived that is consistent with the Chapman-Enskog approach to dc conductivity. For a two-component plasma, explicit calculations have been performed in the lowest moment approach. In the Born approximation, expressions are given that allow the determination of $\epsilon(k,\omega)$ in an analytical way. Within numerical accuracy it is shown that general relations such as sum rules are fulfilled. The dc conductivity is obtained in agreement with the Ziman-Faber result.

We performed exploratory calculations to illustrate how the generalized linear response approach works. Obviously an improvement of the results can be obtained if (i) the Born approximation is improved including higher orders of perturbation theory, and (ii) higher moments of the single-particle distribution are taken into account. Both points have been discussed for the limiting case of dc conductivity [3], where a virial expansion of the inverse conductivity was given.

A four-moment approach will be presented in a subsequent paper [11], where also the comparison with the Kubo approach and computer simulations are discussed. Within the approach given here it is also possible to treat the degenerate case. Work in this direction is in progress.

ACKNOWLEDGMENTS

The author is indebted to V. Morosov and A. Wierling for many helpful discussions, and to A. Schnell for his help in performing the computer calculations.

APPENDIX A: GENERALIZED LINEAR RESPONSE THEORY

Generalized linear response theory has been considered in different works [13], see [9] for a recent presentation. We give here briefly the main ideas to construct the nonequilibrium statistical operator $\rho(t) = \rho_{\rm rel}(t) + \rho_{\rm irrel}(t)$ using the density matrix approach [14]. Characterizing the nonequilibrium state of the system by the mean values $\langle B_n(\vec{r})\rangle^t$ of a set of relevant observables $\{B_n(\vec{r})\}$, from the maximum entropy principle the generalized Gibbs state

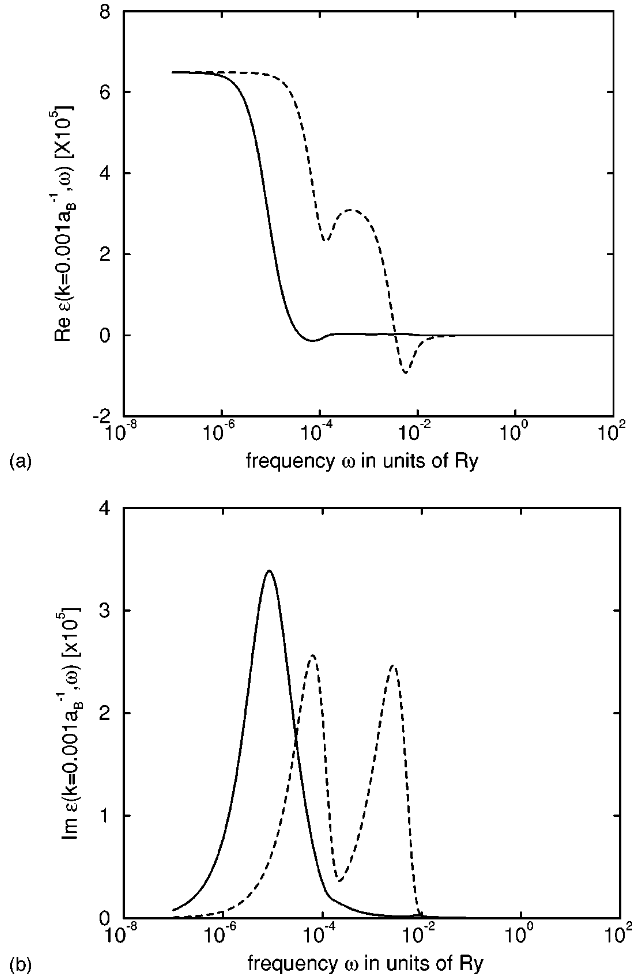

FIG. 4. The same as Fig. 1 for $k=0.001/a_B$.

$$\rho_{\rm rel}(t)=\exp\bigg[-\Phi(t)-\beta H+\beta\sum_c \mu_c N_c$$
$$+\beta\sum_n \int d^3r\,\phi_n(\vec r,t)B_n(\vec r)\bigg] \tag{A1}$$

follows, where $\Phi(t)$ is the Massieu-Planck function. The

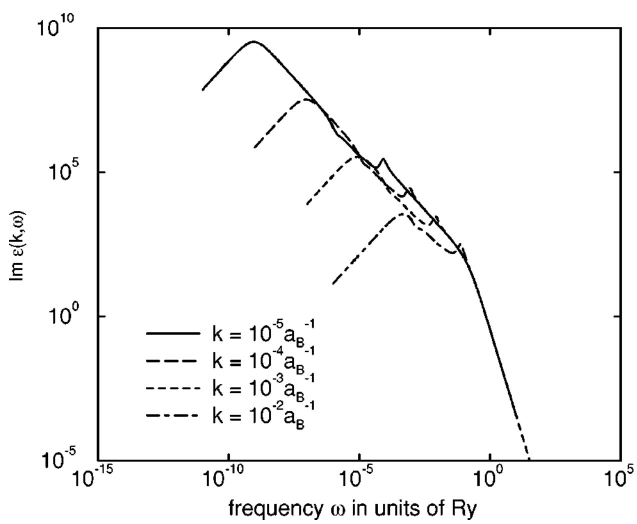

FIG. 5. Im $\epsilon(k,\omega)$ as function of ω for different k.

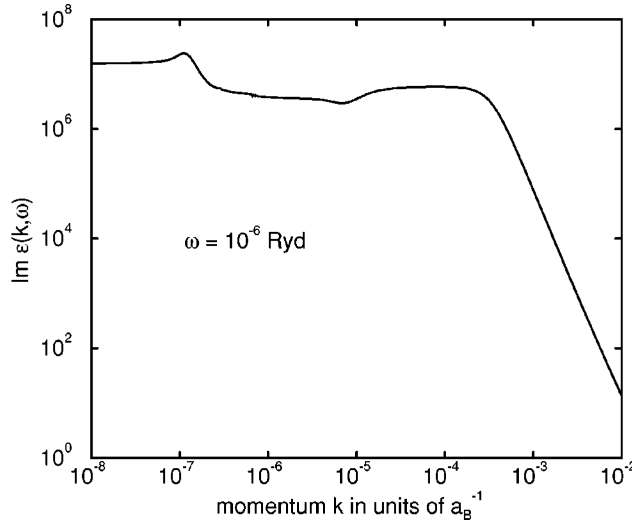

FIG. 6. Im $\epsilon(k,\omega)$ as a function of k for $\omega=0.000\,001$ Ry/$\hbar$.

thermodynamic parameters (Lagrange multipliers) $\phi_n(\vec r,t)$ are determined by the self-consistency conditions

$$\mathrm{Tr}\{B_n(\vec r)\rho_{\rm rel}(t)\}=\langle B_n(\vec r)\rangle^t \tag{A2}$$

and will be evaluated within linear response theory below.

The relevant statistical operator (A1) does not solve the von Neumann equation, but it can serve to formulate the correct boundary conditions to obtain the retarded solution of the von Neumann equation. Using Abel's theorem, the irrelevant part of the nonequilibrium statistical operator [9] is found with the help of the time evolution operator $U(t,t')$,

$$i\hbar\frac{\partial}{\partial t}U(t,t')=H_{\rm tot}(t)U(t,t'),\quad U(t',t')=1\ , \tag{A3}$$

as

$$\rho_{\rm irrel}(t)=-\int_{-\infty}^{t}dt'\,e^{-\eta(t-t')}U(t,t')$$
$$\times\left\{\frac{i}{\hbar}[H_{\rm tot}(t'),\rho_{\rm rel}(t')]+\frac{\partial}{\partial t'}\rho_{\rm rel}(t')\right\}U(t',t), \tag{A4}$$

where the limit $\eta\to 0$ has to be taken after the thermodynamic limit. The self-consistency conditions (A2) which determine the Lagrange multipliers take the form

$$\mathrm{Tr}\{B_n(\vec r)\rho_{\rm irrel}(t)\}=0. \tag{A5}$$

For a weak external field $U_{\rm ext}$, the system remains near thermal equilibrium described by ρ_0 [Eq. (4)]. Expanding the nonequilibrium statistical operator up to first order in $U_{\rm ext}$ and $\phi_n(\vec r,t)$, it is convenient to use the Fourier representation so that

$$\int d^3r\,\phi_n(\vec r,t)B_n(\vec r)=\phi_n(\vec k,\omega)e^{-i\omega t}B_n^\dagger+\text{c.c.} \tag{A6}$$

with

$$\phi_n(\vec{r},t)=e^{i(\vec{k}\cdot\vec{r}-\omega t)}\phi_n(\vec{k},\omega), \quad B_n=\int d^3r B_n(\vec{r})e^{-i\vec{k}\cdot\vec{r}}. \tag{A7}$$

Up to first order, the contributions to $\rho(t)$ are

$$\rho_{\rm rel}(t)=\rho_0+e^{-i\omega t}\int_0^\beta d\tau \sum_n B_n^\dagger(i\hbar\tau)\phi_n(\vec{k},\omega)\rho_0+\text{c.c.} \tag{A8}$$

and, applying the Kubo identity

$$[A,\rho_0]=\int_0^\beta d\tau e^{-\tau H}[H,A]e^{\tau H}\rho_0, \tag{A9}$$

we find

$$\rho_{\rm irrel}(t)=-\int_{-\infty}^t dt' e^{-\eta(t-t')}e^{-i\omega t'}\int_0^\beta d\tau \times\Bigg\{\sum_{c,p} e_c\dot{n}^c_{p,-k}(t'-t+i\hbar\tau)U_{\rm ext}(\vec{k},\omega) + \sum_n [\dot{B}_n^\dagger(t'-t+i\hbar\tau)-i\omega B_n^\dagger \times(t'-t+i\hbar\tau)]\phi_n(\vec{k},\omega)\Bigg\}\rho_0+\text{c.c.} \tag{A10}$$

Inserting this result in the self-consistency conditions (A5) we get the response equations

$$-\langle B_m;A\rangle_{\omega+i\eta}U_{\rm ext}(\vec{k},\omega)=\langle B_m;C\rangle_{\omega+i\eta} \tag{A11}$$

with the correlation functions defined by Eq. (14), $A=ik\Omega_0 J_k=\Sigma_{c,p}e_c\dot{n}^c_{p,k}$, and $C=\Sigma_n[\dot{B}_n+i\omega B_n]\phi_n^*(\vec{k},\omega)$.

To make the relation between the response equations (A11) and the Boltzmann equation more close, see [3], we introduce the "stochastic" part of forces applying partial integrations according to Eq. (15), so that Eq. (A11) can be rewritten as

$$\begin{aligned}-ik\Omega_0(B_m;J_k)U_{\rm eff}(k,\omega)&=\frac{(B_m;J_k)+\langle\dot{B}_m;J_k\rangle_{\omega+i\eta}-\langle\dot{B}_m;J_k\rangle_{\omega+i\eta}}{\langle B_m;J_k\rangle_{\omega+i\eta}}\langle B_m;C\rangle_{\omega+i\eta}\\ &=(B_m;C)+\left\langle\left[\dot{B}_m-\frac{\langle\dot{B}_m;J_k\rangle_{\omega+i\eta}}{\langle B_m;J_k\rangle_{\omega+i\eta}}B_m\right];\left[C-\frac{\langle B_m;C\rangle_{\omega+i\eta}}{\langle B_m;J_k\rangle_{\omega+i\eta}}J_k\right]\right\rangle_{\omega+i\eta}.\end{aligned} \tag{A12}$$

We find the following form for the response equations:

$$-ik\Omega_0 M_{m0}U_{\rm ext}(k,\omega)=\sum_n M_{mn}\phi_n(k,\omega) \tag{A13}$$

with $M_{m0}=(B_m;J_k)$ and

$$M_{mn}=(B_m;[\dot{B}_n+i\omega B_n]) +\left\langle\left[\dot{B}_m-\frac{\langle\dot{B}_m;J_k\rangle_{\omega+i\eta}}{\langle B_m;J_k\rangle_{\omega+i\eta}}B_m\right];[\dot{B}_n+i\omega B_n]\right\rangle_{\omega+i\eta}. \tag{A14}$$

The system of equations (A13) can be solved applying the Cramers rule. Then, the response parameters are represented as a ratio of two determinants.

With the solutions ϕ_n, the explicit form of $\rho(t)$ is known, and we can evaluate mean values of arbitrary observables. In particular, we are interested in the evaluation of $\langle J_k\rangle^t \exp(i\omega t)$ to calculate the polarization function (10) using Eqs. (A8) and (A10),

$$\langle J_k\rangle^t e^{i\omega t}=\beta\sum_n \{(J_k;B_n)-\langle J_k;[\dot{B}_n+i\omega B_n]\rangle_{\omega+i\eta}\}\phi_n(\vec{k},\omega) -ik\Omega_0\beta\langle J_k;J_k\rangle_{\omega+i\eta}U_{\rm ext}(\vec{k},\omega). \tag{A15}$$

If J_k can be represented by a linear combination of the relevant observables $\{B_n\}$, we can directly use the self-consistency conditions (A2) and have

$$\langle J_k\rangle^t e^{i\omega t}=\text{Tr}[J_k\rho_{\rm rel}(t)]e^{i\omega t}. \tag{A16}$$

Comparing with Eq. (A15) we see that the remaining terms on the right-hand side of Eq. (A15) compensate due to the response equations (A11). After expanding $\rho_{\rm rel}(t)$ up to first order in $\phi_n(\vec{k},\omega)$, Eq. (A8), we have

$$\langle J_k\rangle^t e^{i\omega t}=\beta\sum_n (J_k;B_n)\phi_n(\vec{k},\omega). \tag{A17}$$

Inserting the solutions for ϕ_n in the form of determinants, we get for the polarization function (10) with $M_{0n}(k,\omega)=(J_k;B_n)$ the result

$$\Pi(k,\omega)=i\epsilon(k,\omega)\beta\Omega_0\frac{k^2}{\omega}\times\begin{vmatrix}0 & M_{0n}(k,\omega)\\ M_{m0}(k,\omega) & M_{mn}(k,\omega)\end{vmatrix}\Bigg/ |M_{mn}(k,\omega)|. \tag{A18}$$

Specifying to only one relevant observable $B_n=J_k$, the result (12) for the polarization function follows.

APPENDIX B: EVALUATION OF THE COLLISION TERM IN BORN APPROXIMATION

Let us first consider the lowest order of perturbation theory where the correlation functions are immediately evaluated using Wick's theorem. We find

$$(n^d_{p,k};n^c_{p',k})=\hat{f}^c_{p,k}\delta_{pp'}\delta_{cd}, \tag{B1}$$

$$\langle n^d_{p,k};n^c_{p',k}\rangle_{\omega+i\eta}=(\eta-i\omega+i\hbar p_z k/m_c)^{-1}\hat{f}^c_{p,k}\delta_{pp'}\delta_{cd},$$

with

$$\hat{f}^c_{p,k}=(\beta\hbar^2 p_z k/m_c)^{-1}(f^c_{p-k/2}-f^c_{p+k/2}). \tag{B2}$$

Note that $\lim_{k\to 0}\hat{f}^c_{p,k}=f^c_p=\{\exp[\beta(E^c_p-\mu_c)]+1\}^{-1}$. In the classical limit where the Fermi function can be replaced by the Maxwell distribution, we have to lowest order in the Coulomb interaction

$$(J_k;J_k)^{(0)}=\frac{k_BT}{\Omega_0}\sum_c\frac{e_c^2}{m_c}n_c, \tag{B3}$$

$$\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta}=-i\frac{\omega}{k^2}\frac{1}{\Omega_0}\sum_c e_c^2n_c[1+z_cD(z_c)], \tag{B4}$$

with $z_c=\omega/k\sqrt{m_c/2k_BT}$ and the Dawson integral

$$D(z)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-x^2}\frac{dx}{x-z-i\eta}. \tag{B5}$$

Furthermore, we have

$$\begin{aligned}\langle \dot{J}_k;J_k\rangle^{(0)}_{\omega+i\eta}&=-\frac{k_BT}{\Omega_0}\sum_c\frac{e_c^2}{m_c}n_c\\&\quad-\frac{\omega^2}{k^2}\frac{1}{\Omega_0}\sum_c e_c^2n_c[1+z_cD(z_c)]\\&=-\langle J_k;\dot{J}_k\rangle^{(0)}_{\omega+i\eta},\end{aligned} \tag{B6}$$

$$\begin{aligned}\langle \dot{J}_k;\dot{J}_k\rangle^{(0)}_{\omega+i\eta}&=-i\omega\frac{k_BT}{\Omega_0}\sum_c\frac{e_c^2}{m_c}n_c\\&\quad-i\frac{\omega^3}{k^2}\frac{1}{\Omega_0}\sum_c e_c^2n_c[1+z_cD(z_c)],\end{aligned} \tag{B7}$$

so that from Eq. (12) the random phase approximation (RPA)

$$\Pi^{(0)}(k,\omega)=-\beta\sum_c e_c^2n_c[1+z_cD(z_c)] \tag{B8}$$

is obtained. The prefactor $\epsilon(k,\omega)$ disappears taking into account only irreducible diagrams for $\Pi(k,\omega)$. The corresponding RPA dielectric function (6) describes the collisionless plasma.

To include collisions, we have to consider higher orders of the interaction. In the numerator of Eq. (12), the higher order expansion for $(J_k;J_k)$ leads to the replacement of the occupation numbers f^c_p for the free fermion gas by the occupation numbers in an interacting fermion gas. These self-energy corrections in the Born approximation can be given as a shift of the single-particle energies and can be replaced by a shift of the chemical potential, see [3]. They do not describe collision effects and will not be considered here.

Collision terms arise from the time correlation functions such as $\langle \dot{J}_k;\dot{J}_k\rangle_{\omega+i\eta}$ in Eq. (13). The force term $\dot{J}_k$ already contains the interaction $V_{cc'}(q)$ according to Eq. (25), so that a collision term in the Born approximation is obtained if the corresponding correlation function containing the creation and annihilation operators $a^\dagger,a$ is evaluated in zeroth order with respect to the interaction. To extract also the collision terms in the Born approximation from time correlation functions such as $\langle J_k;\dot{J}_k\rangle_{\omega+i\eta}$, we use the relations

$$\begin{aligned}\langle n^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}&=(\eta-i\omega+i\hbar p_zk/m_c)^{-1}\\&\quad\times[(n^c_{p,k};v^d_{p',k})+\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}],\end{aligned} \tag{B9}$$

$$\begin{aligned}\langle v^c_{p,k};n^d_{p',k}\rangle_{\omega+i\eta}&=(\eta-i\omega+i\hbar p'_zk/m_d)^{-1}\\&\quad\times[(v^c_{p,k};n^d_{p',k})-\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}],\end{aligned} \tag{B10}$$

which can be proven by partial integration (15). Collecting all terms of the form $\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}$ which contribute to collisions in the Born approximation, we find from Eq. (13)

$$\begin{aligned}M_{JJ}&=\frac{[(J_k;J_k)^{(0)}]^2}{\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta}}+\sum_{cd,pp'}\frac{\hbar^2}{\Omega_0^2}\frac{e_ce_d}{m_cm_d}p_zp'_z\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}\\&\quad\times\left\{-1+\frac{(J_k;J_k)^{(0)}}{\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta}}\left[\frac{1}{\eta-i\omega+i\hbar p'_zk/m_d}\right.\right.\\&\quad\left.\left.+\frac{1}{\eta-i\omega+i\hbar p_zk/m_c}\right]\right\}=M^{(0)}_{JJ}+M^{(1)}_{JJ}.\end{aligned} \tag{B11}$$

Introducing according to Eqs. (B3) and (B4)

$$R=\frac{(J_k;J_k)^{(0)}}{\langle J_k;J_k\rangle^{(0)}_{\omega+i\eta}}=ik_BT\,\frac{k^2}{\omega}\frac{\Sigma_c e_c^2 n_c/m_c}{\Sigma_c e_c^2 n_c[1+z_cD(z_c)]}, \tag{B12}$$

we have

$$M^{(0)}_{JJ}=R(J_k;J_k)^{(0)}, \tag{B13}$$

$$M^{(1)}_{JJ}=\frac{\hbar^2}{\Omega_0^2}\sum_{cd,pl}\frac{e_ce_d}{m_cm_d}p_zl_z\langle v^c_{p,k};v^d_{l,k}\rangle_{\omega+i\eta}\times\left\{-1+R\left[\frac{1}{\eta-i\omega+i\hbar p_zk/m_c}+\frac{1}{\eta-i\omega+i\hbar l_zk/m_d}\right]\right\}. \tag{B14}$$

Dropping single-particle exchange terms that can be justified for the Coulomb interaction in the low-density limit, we find the collision term in the Born approximation by using Wick's theorem,

$$\begin{aligned}\langle v^c_{p,k};v^d_{p',k}\rangle_{\omega+i\eta}=&\frac{\pi}{\hbar}\sum_{c'p''q}\frac{\exp(-\beta\hbar\omega)-1}{\beta\hbar\omega}V_{cc'}(q)f^{c'}_{p''+q}(1-f^{c'}_{p''})\\ &\times\{f^c_{p-k/2-q}(1-f^c_{p+k/2})\,\delta(E^c_{p-k/2-q}+E^{c'}_{p''+q}-E^c_{p+k/2}-E^{c'}_{p''}+\hbar\omega)\\ &\times[V_{cc'}(-q)\,\delta_{cd}(\delta_{p',p-q}-\delta_{p',p})+V_{c'c}(k+q)\,\delta_{c'd}(\delta_{p',p''+k/2+q}-\delta_{p',p''-k/2})]\\ &-f^c_{p-k/2}(1-f^c_{p+k/2+q})\,\delta(E^c_{p-k/2}+E^{c'}_{p''+q}-E^c_{p+k/2+q}-E^{c'}_{p''}+\hbar\omega)\\ &\times[V_{cc'}(-q)\,\delta_{cd}(\delta_{p',p}-\delta_{p',p+q})+V_{c'c}(k+q)\,\delta_{c'd}(\delta_{p',p''+k/2+q}-\delta_{p',p''-k/2})]\}.\end{aligned} \tag{B15}$$

For small $k,\ \omega$ we have with Eqs. (B14) and (B15)

$$\begin{aligned}M^{(1)}_{JJ}=2\,\frac{\pi\hbar}{\Omega_0^2}\sum_{lpq}&V^2_{ei}(q)f^e_pf^i_l\delta(E^e_{p+q}+E^i_{l-q}-E^e_p-E^i_l)q_z\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)\left\{\left(\frac{e_e}{m_e}p_z+\frac{e_i}{m_i}l_z\right)\right.\\ &\left.-2R\left(\frac{p_z}{i\hbar kp_z/m_e-i\omega+\eta}\,\frac{e_e}{m_e}+\frac{l_z}{i\hbar kl_z/m_i-i\omega+\eta}\,\frac{e_i}{m_i}\right)\right\}.\end{aligned} \tag{B16}$$

The further evaluation is done introducing total and relative momenta $\vec{P}=\vec{p}+\vec{l},\ \vec{p}\,'=(m_i\vec{p}-m_e\vec{l})/M_{ei},\ \vec{p}\,''=\vec{p}\,'+\vec{q}$, $M_{ei}=m_e+m_i,\ \mu_{ei}^{-1}=m_e^{-1}+m_i^{-1}$. Inserting the distribution functions $f^c_p=n_c(2\pi\hbar^2/m_ck_BT)^{3/2}\exp[-\hbar^2p^2/(2m_ck_BT)]$ and the screened potential $V_{ei}(q)=e_ee_i/(\epsilon_0\Omega_0(q^2+\kappa^2))$, we obtain

$$\begin{aligned}M^{(1)}_{JJ}=&2\frac{\hbar\pi}{\Omega_0}\,\frac{e_e^2e_i^2}{\epsilon_0^2}n_e\left(\frac{2\pi\hbar^2}{m_ek_BT}\right)^{3/2}n_i\left(\frac{2\pi\hbar^2}{m_ik_BT}\right)^{3/2}\frac{1}{(2\pi)^9}\,\frac{2\mu_{ei}}{\hbar^2}\int d^3P\int d^3p'\int d^3p''\\ &\times e^{-\frac{\hbar^2P^2}{2M_{ei}k_BT}}e^{-\frac{\hbar^2p'^2}{2\mu_{ei}k_BT}}\delta(p'^2-p''^2)\frac{1}{[(\vec{p}\,'-\vec{p}\,'')^2+\kappa^2]^2}(p''_z-p'_z)\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)\\ &\times\left\{p'_z\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)-2R\,\frac{M_{ei}\omega}{i\hbar^2k^2}\left(\frac{e_e}{P_z+\frac{M_{ei}}{m_e}p'_z-\frac{M_{ei}\omega}{\hbar k}-i\eta}+\frac{e_i}{P_z-\frac{M_{ei}}{m_i}p'_z-\frac{M_{ei}\omega}{\hbar k}-i\eta}\right)\right\}.\end{aligned} \tag{B17}$$

Furthermore, we introduce dimensionless variables $\hbar P(2M_{ei}k_BT)^{1/2},\ \hbar p'(2\mu_{ei}k_BT)^{1/2},\ \lambda=(\hbar^2\kappa^2)/(4\mu_{ei}k_BTp'^2)+1$, spherical coordinates $\vec{p}\,'=(p'(1-c^2)^{1/2},0,p'c)$, $\vec{p}\,''=(p''(1-z^2)^{1/2}\cos\phi,p''(1-z^2)^{1/2}\sin\phi,p''z)$, and perform the integral over ϕ according to

$$\int_0^{2\pi}d\phi\frac{1}{[\lambda-cz-\sqrt{1-c^2}\sqrt{1-z^2}\,\cos\phi]^2}=2\pi\frac{\lambda-cz}{(\lambda^2-1+c^2-2\lambda cz+z^2)^{3/2}} \tag{B18}$$

so that

$$M_{JJ}^{(1)}=\frac{1}{\Omega_0}n_e n_i \frac{e_e^2 e_i^2}{\epsilon_0^2}\left(\frac{\mu_{ei}}{(2\pi)^3 k_B T}\right)^{1/2}\frac{1}{2}\int_0^\infty \frac{1}{p'}dp' e^{-p'^2}\int_{-1}^{1}dc\int_{-1}^{1}dz\,\frac{1}{\pi^{3/2}}\int d^3P e^{-P^2}\frac{\lambda-cz}{(\lambda^2-1+c^2-2\lambda cz+z^2)^{3/2}}(z-c)$$
$$\times\left\{p'^2 c\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)^2+iRp'\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)\frac{\omega}{k_B T k^2}\sqrt{\frac{M_{ei}}{\mu_{ei}}}\left[\frac{e_e}{P_z+\sqrt{\frac{m_i}{m_e}}p'c-\frac{\omega}{k}\sqrt{\frac{M_{ei}}{2k_B T}}}+\frac{e_i}{P_z-\sqrt{\frac{m_e}{m_i}}p'c-\frac{\omega}{k}\sqrt{\frac{M_{ei}}{2k_B T}}}\right]\right\}. \tag{B19}$$

Now, the integrals over z and P can be performed. Using

$$\int_{-1}^{1}dz\,\frac{\lambda-cz}{(\lambda^2-1+c^2-2\lambda cz+z^2)^{3/2}}(z-c)=c\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right), \tag{B20}$$

and the definition of the Dawson integral (29) to perform the integral over P_z, we finally find after integrating the transverse components of $\vec{P}$

$$M_{JJ}^{(1)}=\frac{1}{\Omega_0}n_e n_i \frac{e_e^2 e_i^2}{\epsilon_0^2}\left(\frac{\mu_{ei}}{2k_B T}\right)^{1/2}\frac{1}{4\pi^{3/2}}\int_0^\infty dp\, e^{-p^2}\left(\ln\frac{\lambda-1}{\lambda+1}+\frac{2}{\lambda+1}\right)\left\{\frac{2}{3}p\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)^2\right.$$
$$\left.+iR\left(\frac{e_e}{m_e}-\frac{e_i}{m_i}\right)\frac{\omega}{k_B T k^2}\sqrt{\frac{M_{ei}}{\mu_{ei}}}\int_{-1}^{1}dc\,c\left[e_e D\left(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp\right)+e_i D\left(z_{ei}+\sqrt{\frac{m_e}{m_i}}cp\right)\right]\right\} \tag{B21}$$

with $z_{ei}=\omega/k\sqrt{M_{ei}/2k_B T}$. Together with Eqs. (B13) and (B3), this result can be inserted in expression (12) to evaluate $\Pi(k,\omega)$.

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